NUCLEONICA:
Decay Engine

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Decay Engine

Some theory

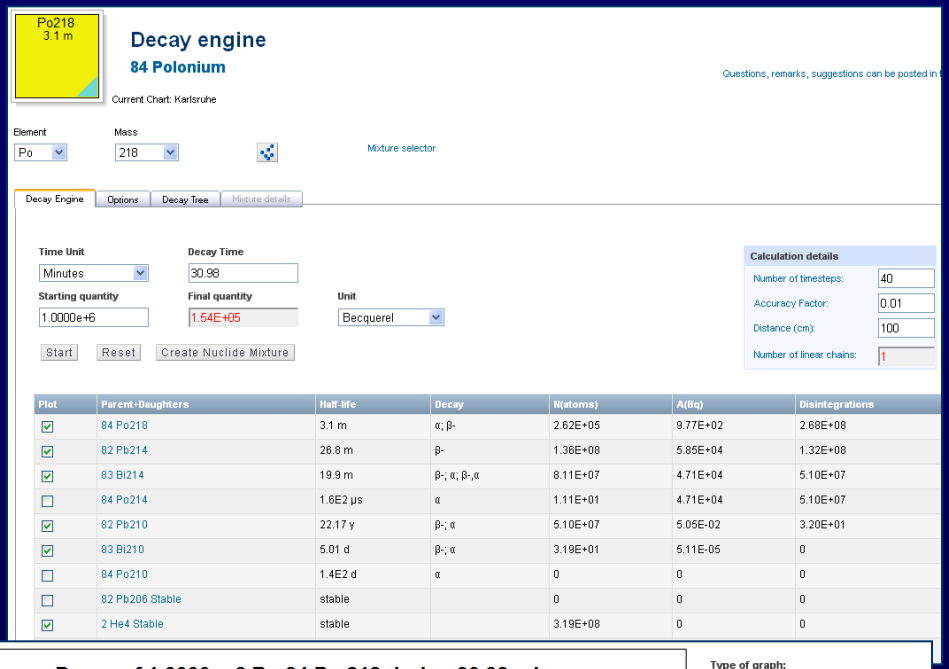
Launching Decay Engine

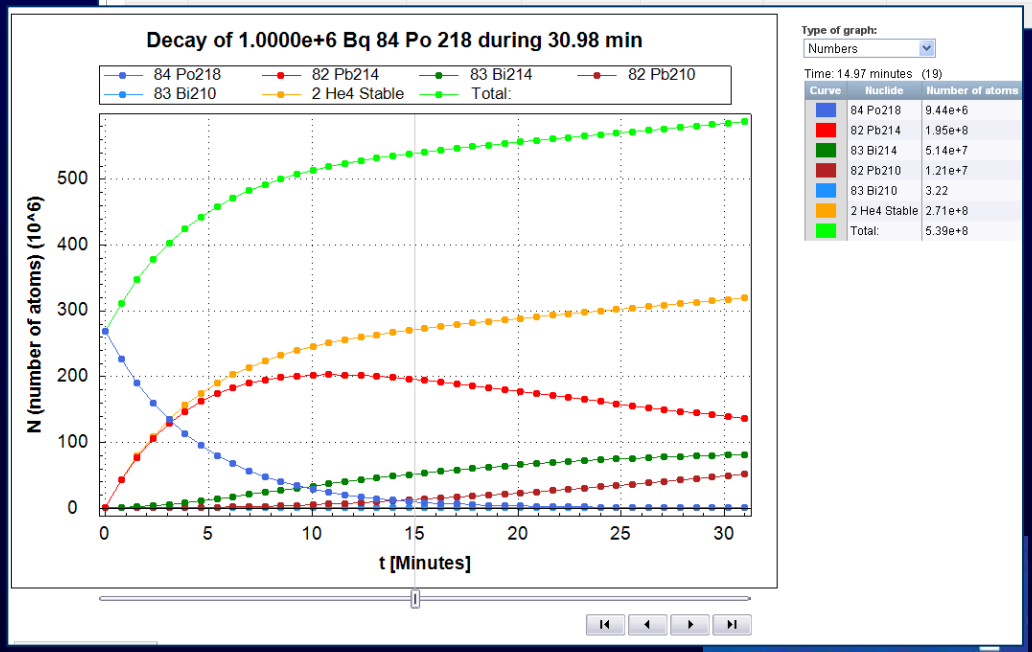
Performing calculation with default settings

Exploring calculation results

Selecting options

Plotting results





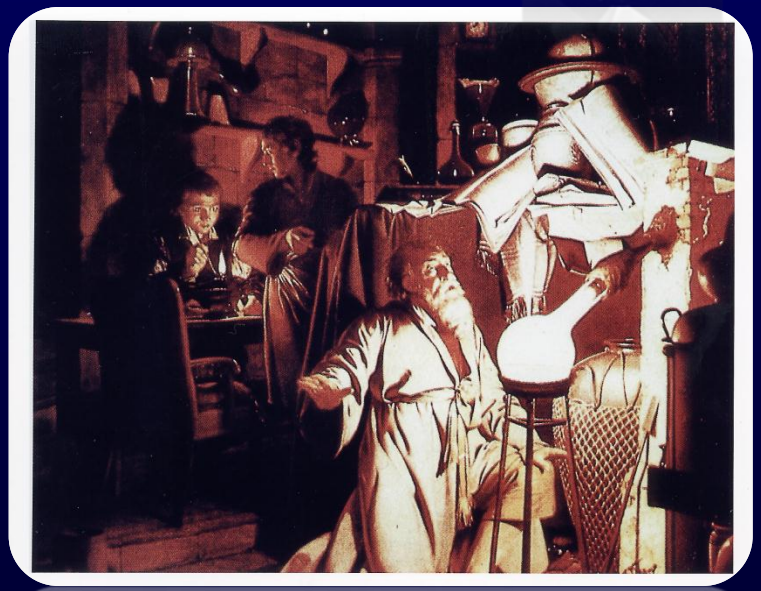
Modern Alchemy: Discovery of transmutation, (Soddy 1901)

In 1901, twenty-four year-old chemist Frederick Soddy and Ernest Rutherford were attempting to identify a mysterious gas that wafted from samples of radioactive thorium oxide. They suspected that this gas—they called it an "emanation"—held a key to the recently discovered phenomenon of radioactivity. Soddy had passed the puzzling gas over a series of powerful chemical reagents, heated white-hot. When no reactions took place, he came to a startling realization. As he told his biographer many years later….

´I remember quite well standing there transfixed as though stunned by the colossal import of the thing and blurting out- or so it seemed at the time: "Rutherford, this is transmutation: the thorium is disintegrating and transmuting itself into argon gas". Rutherford's reply was typically aware of more practical implications,"For Mike's sake, Soddy, don't call it *transmutation*. They'll have our our heads off as alchemists"`

*quoted in *Pioneer*, pp 83-84

Frederic Soddy



Joseph Wright (1734-1797)

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Simple radioactive decay..

Basic equation first identified by Rutherford

$$dQ/dt = -kQ \qquad (1)$$

Q is the number of atoms, k is the decay constant (probability per unit time that a nucleus will decay):

$$k = \ln 2 / \tau$$

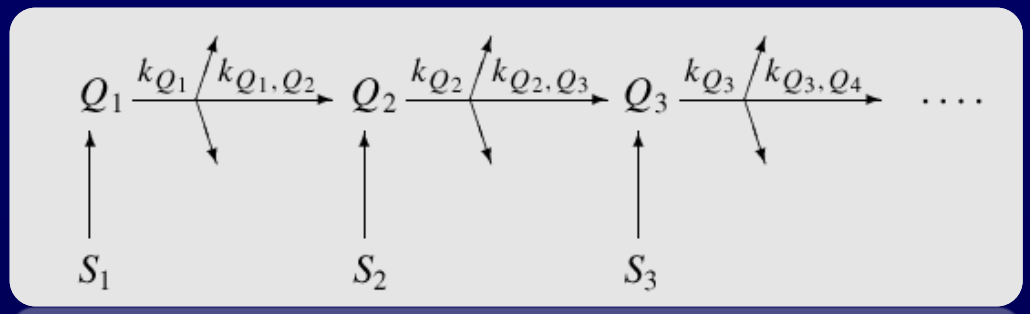
where τ is the half-live. Solution of Eq.1...

$$Q(t) = Q(0)\, e^{-kt} \quad \text{or} \quad Q(t) = Q(0)\, 2^{-t/\tau}$$

Activity..

$$A(t) = k\, Q(t)$$

Successive radioactive decay with branching and source terms



The differential equations governing the above processes:

$$\mathrm{d}Q_1/\mathrm{d}t = S_1 - k_{Q_1} \cdot Q_1 \,,$$
$$\mathrm{d}Q_2/\mathrm{d}t = S_2 + k_{Q_1,Q_2} \cdot Q_1 - k_{Q_2} \cdot Q_2 \,,$$
$$\mathrm{d}Q_i/\mathrm{d}t = S_i + k_{Q_{i-1},Q_i} \cdot Q_{i-1} - k_{Q_i} \cdot Q_i \,,$$
$$\mathrm{d}Q_n/\mathrm{d}t = S_n + k_{Q_{n-1},Q_n} \cdot Q_{n-1} - k_{Q_n} \cdot Q_i \,,$$


H. Bateman

Mr Bateman, Solution of a system of differential equations, etc. 423

The solution of a system of differential equations occurring in the theory of radio-active transformations. By H. Bateman, M.A., Trinity College.

[*Read* 21 February 1910.]

1. It has been shown by Prof. Rutherford * that the amounts of the primary substance and the different products in a given quantity of radio-active matter vary according to the system of differential equations,

$$\left.\begin{aligned} \frac{dP}{dt} &= -\lambda_1 P \\ \frac{dQ}{dt} &= \lambda_1 P - \lambda_2 Q \\ \frac{dR}{dt} &= \lambda_2 Q - \lambda_3 R \\ \frac{dS}{dt} &= \lambda_3 R - \lambda_4 T \\ &\cdots\cdots\cdots \end{aligned}\right\} \cdots\cdots\cdots\cdots\cdots\cdots (1),$$

denote the number of atoms of the primary
ve products which are present at time t.
as worked out the various cases in which
ducts in addition to the primary substance,
ght as if the results may be extended to any
ithout much labour.
straightforward method is unsymmetrical
the results of the calculations are needed in
which are being carried on in radio-activity
ht it worth while to publish a simple and
of obtaining the required formulæ.
ce a set of auxiliary quantities $p(x)$, $q(x)$,
on a variable x and connected with the
$R(t)$, ... by the equations

$$p(x) = \int_0^\infty e^{-xt} P(t)\,dt, \quad q(x) = \int_0^\infty e^{-xt} Q(t)\,dt \cdots\cdots (2).$$

It is easily seen that

$$\int_0^\infty e^{-xt} \frac{dP}{dt}\,dt = -P(0) + x\int_0^\infty e^{-xt} P(t)\,dt \cdots\cdots (3),$$
$$= -P_0 + xp,$$

* *Radio-activity*, 2nd edition, p. 382.

Exact solution:

$$Q_{\mathrm{n}}(t) = \sum_{i=1}^{i=n} \left[\left(\prod_{j=1}^{j=n-1} k_{j,j+1} \right) \times \sum_{j=i}^{j=n} \left(\frac{Q_i\,(0)\,\mathrm{e}^{-k_j t}}{\prod_{\substack{p=i \\ p\neq j}}^{n} \left(k_p - k_j\right)} + \frac{S_i\left(1 - \mathrm{e}^{-k_j t}\right)}{k_j \prod_{\substack{p=i \\ p\neq j}}^{n} \left(k_p - k_j\right)} \right) \right]$$

For $S_i = 0$:

$$Q_{\mathrm{n}}\,(t) = \prod_{j=1}^{j=n-1} k_{j,j+1} \sum_{j=i}^{j=n} \frac{Q_i\,(0)\,\mathrm{e}^{-k_j t}}{\prod_{\substack{p=i \\ p\neq j}}^{n} \left(k_p - k_j\right)}$$

First few terms...

$$Q_1 = Q_1\,(0)\,\mathrm{e}^{-k_1 t}$$

$$Q_2 = k_{1,2} \left\{ \frac{Q_1\,(0)\,\mathrm{e}^{-k_1 t}}{k_2 - k_1} + \frac{Q_1\,(0)\,\mathrm{e}^{-k_2 t}}{k_1 - k_2} \right\}$$

$$Q_3 = k_{1,2} k_{2,3} \left\{ \frac{Q_1\,(0)\,e^{-k_1 t}}{(k_2 - k_1)\,(k_3 - k_1)} + \frac{Q_1\,(0)\,\mathrm{e}^{-k_2 t}}{(k_1 - k_2)\,(k_3 - k_2)} + \frac{Q_1\,(0)\,\mathrm{e}^{-k_3 t}}{(k_1 - k_3)\,(k_2 - k_3)} \right\}$$

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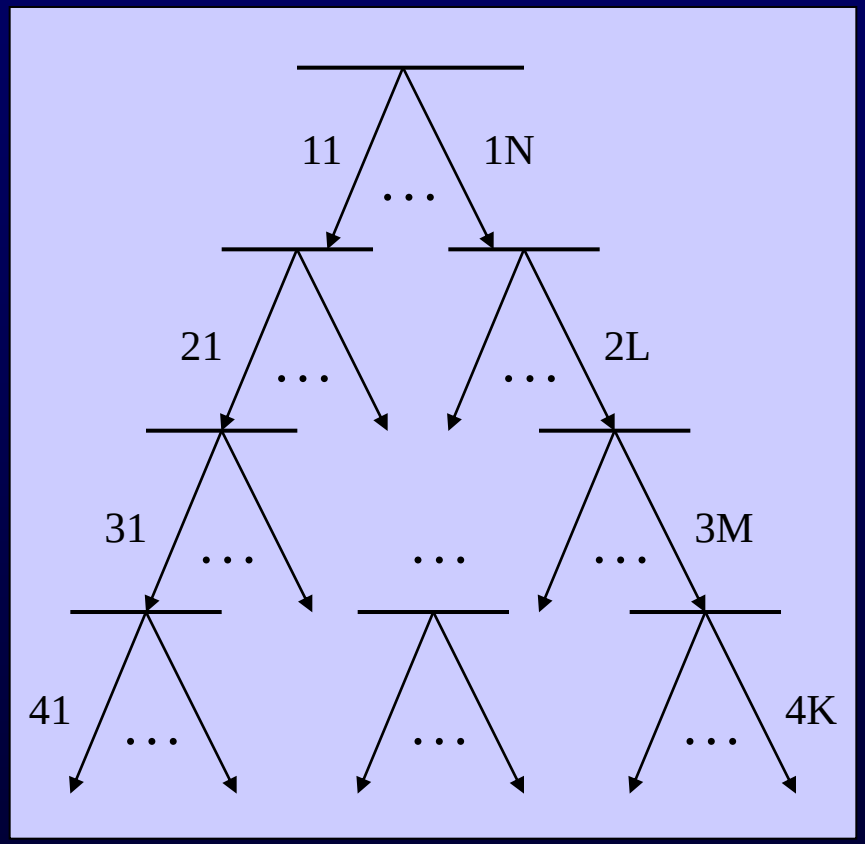
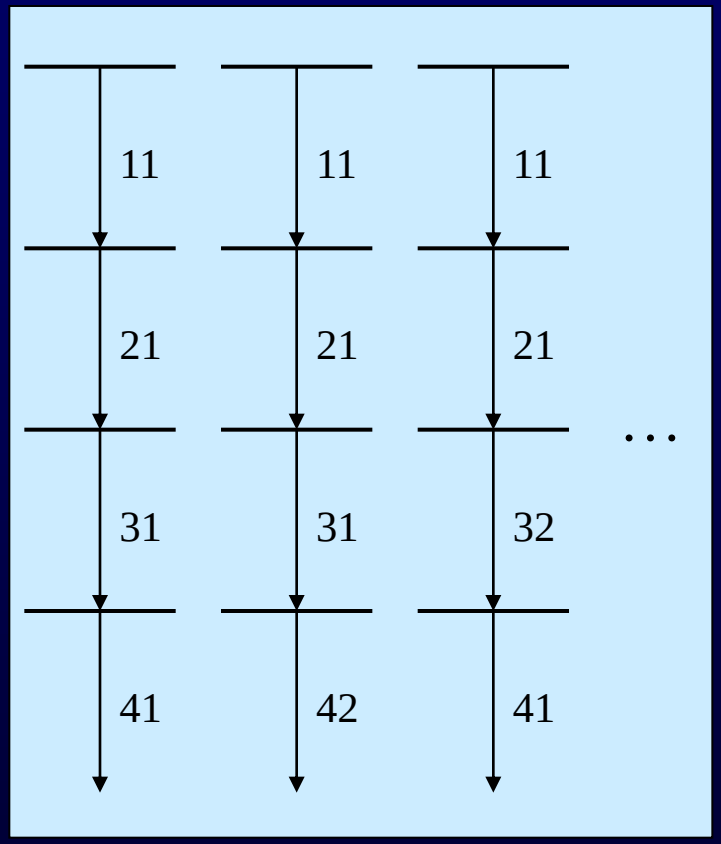
Decay Tree



=>

Linear Chains



$$Prod_1 = BR_{11}BR_{21}BR_{31}BR_{41}$$

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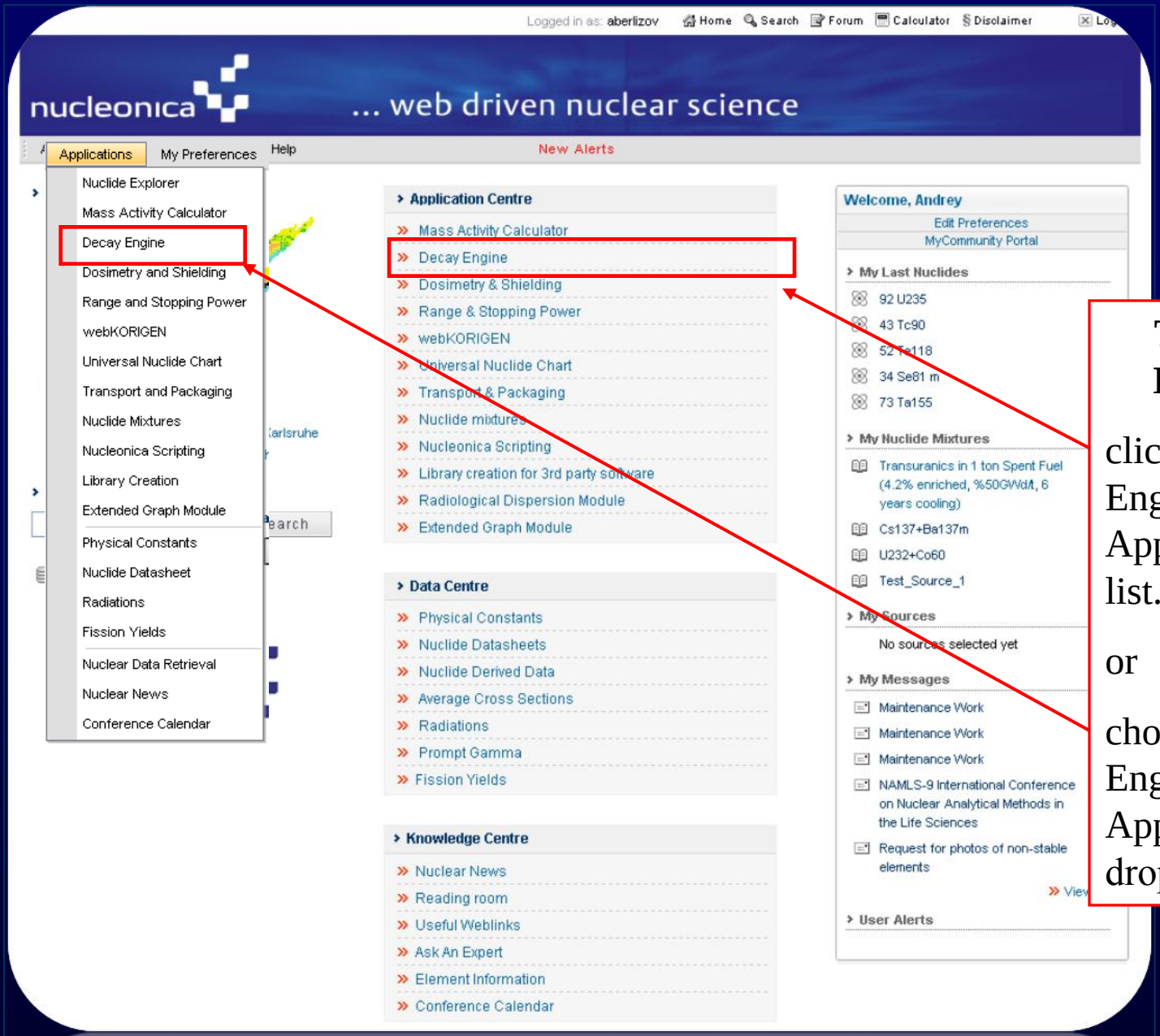

To launch the Decay Engine

click on Decay Engine in the Application Center list.....

or

choose Decay Engine from the Applications dropdown list….

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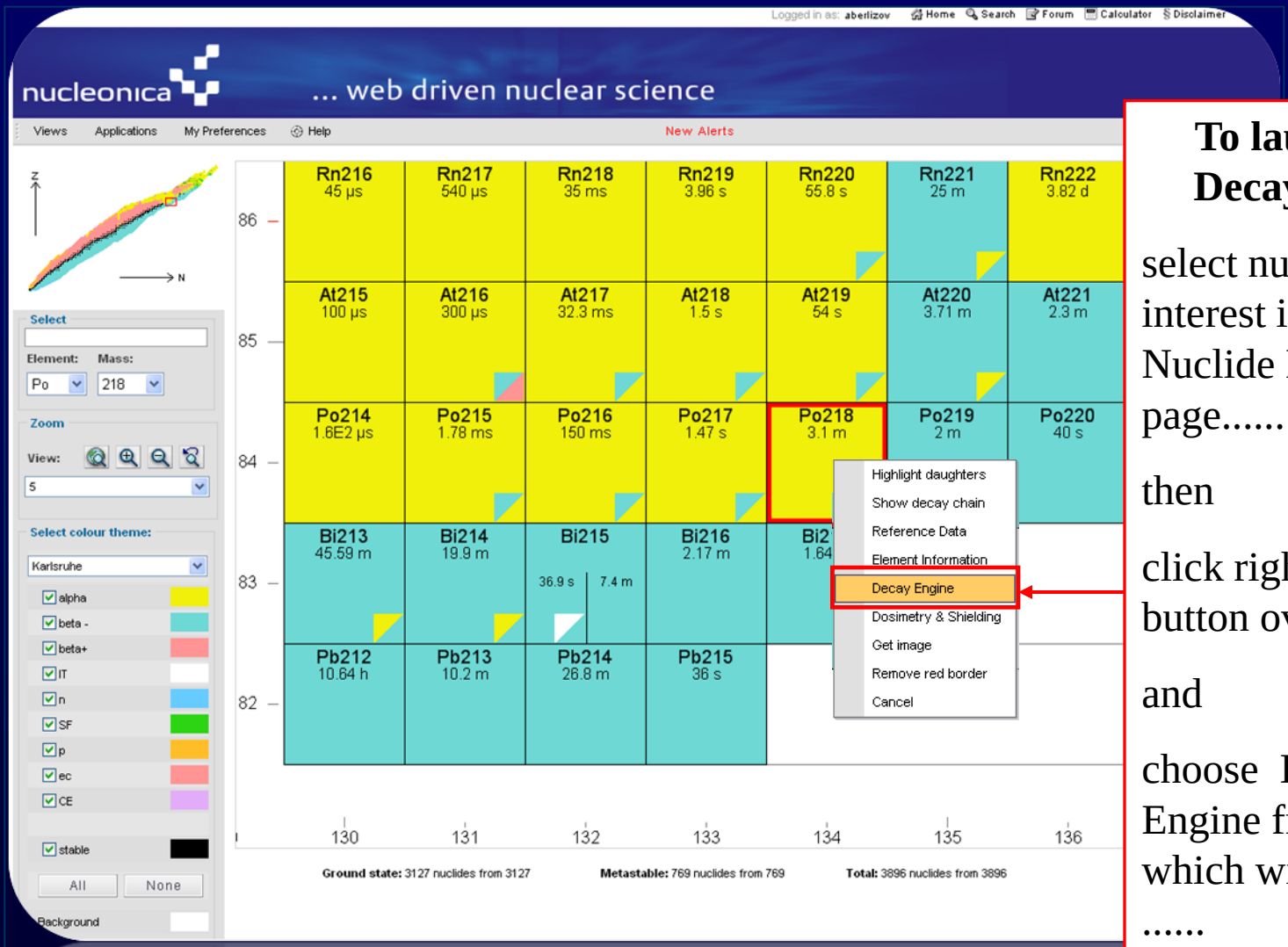

To launch the Decay Engine

select nuclide of interest in the Nuclide Explorer page.....

then

click right mouse button over it .....

and

choose Decay Engine from the list, which will appear .....

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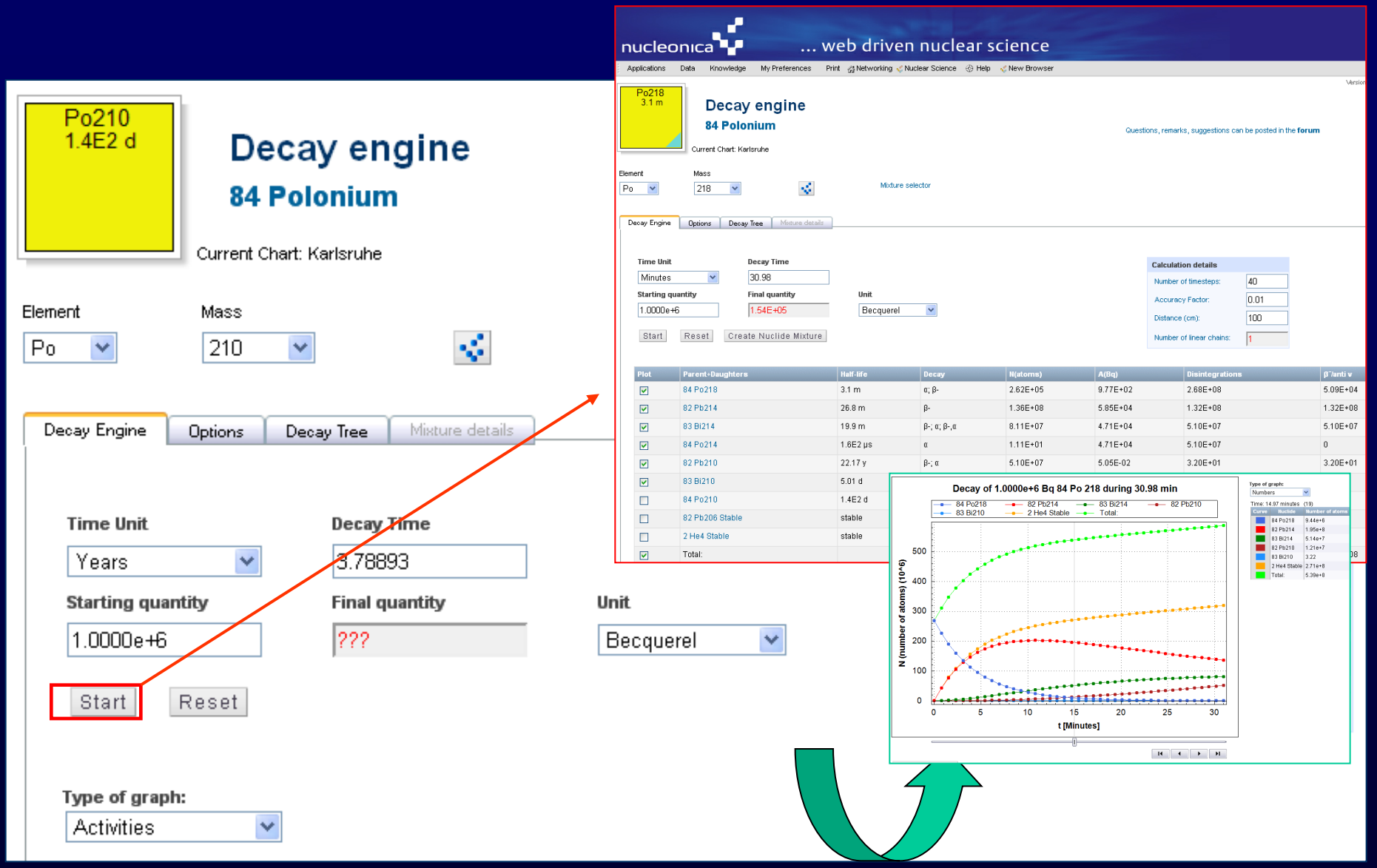

Start a calculation using default values....

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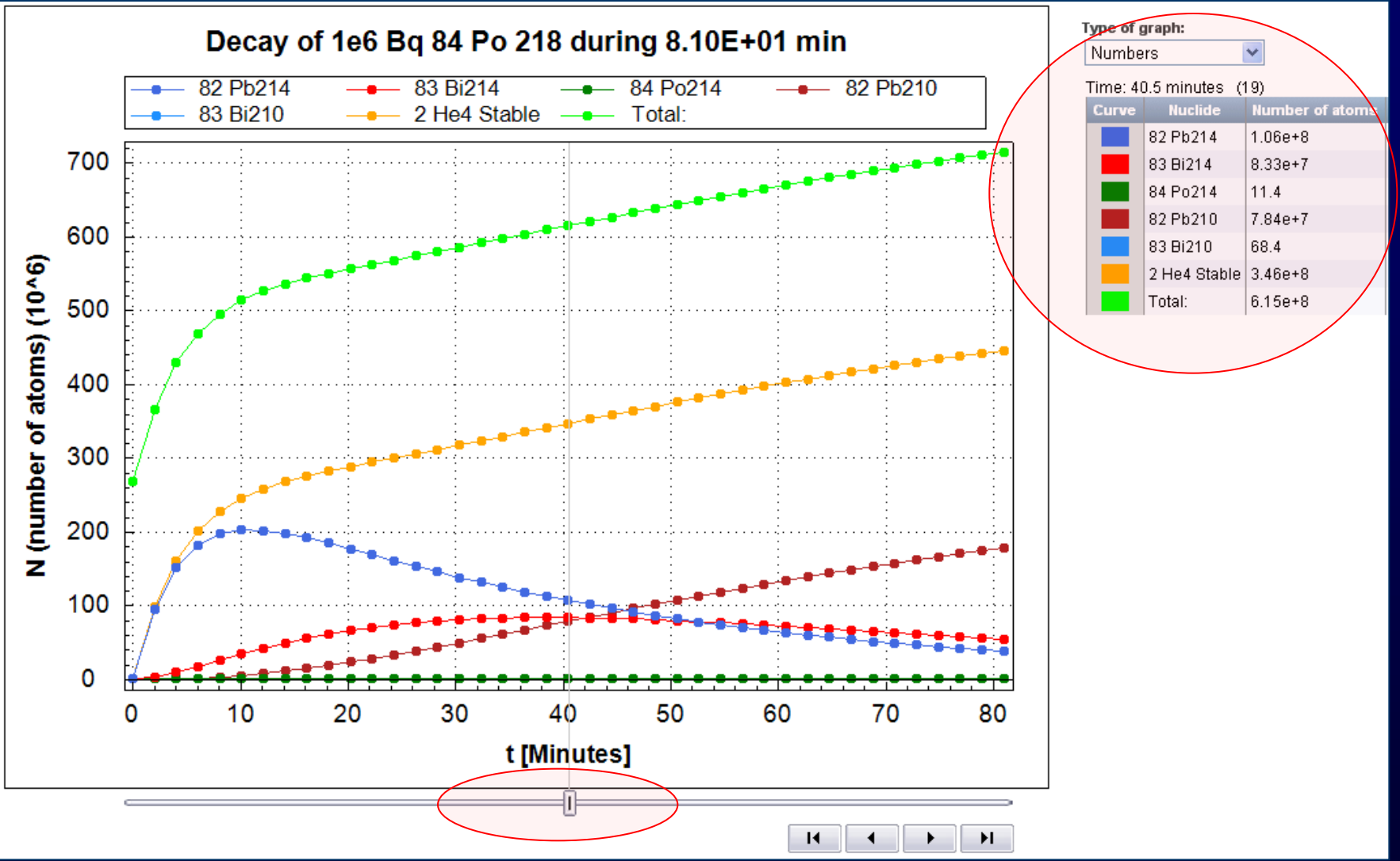
Slider control for easy reading graph values…



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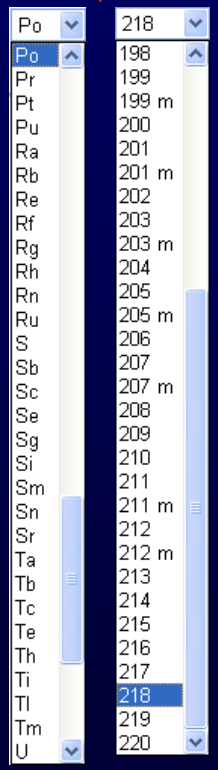
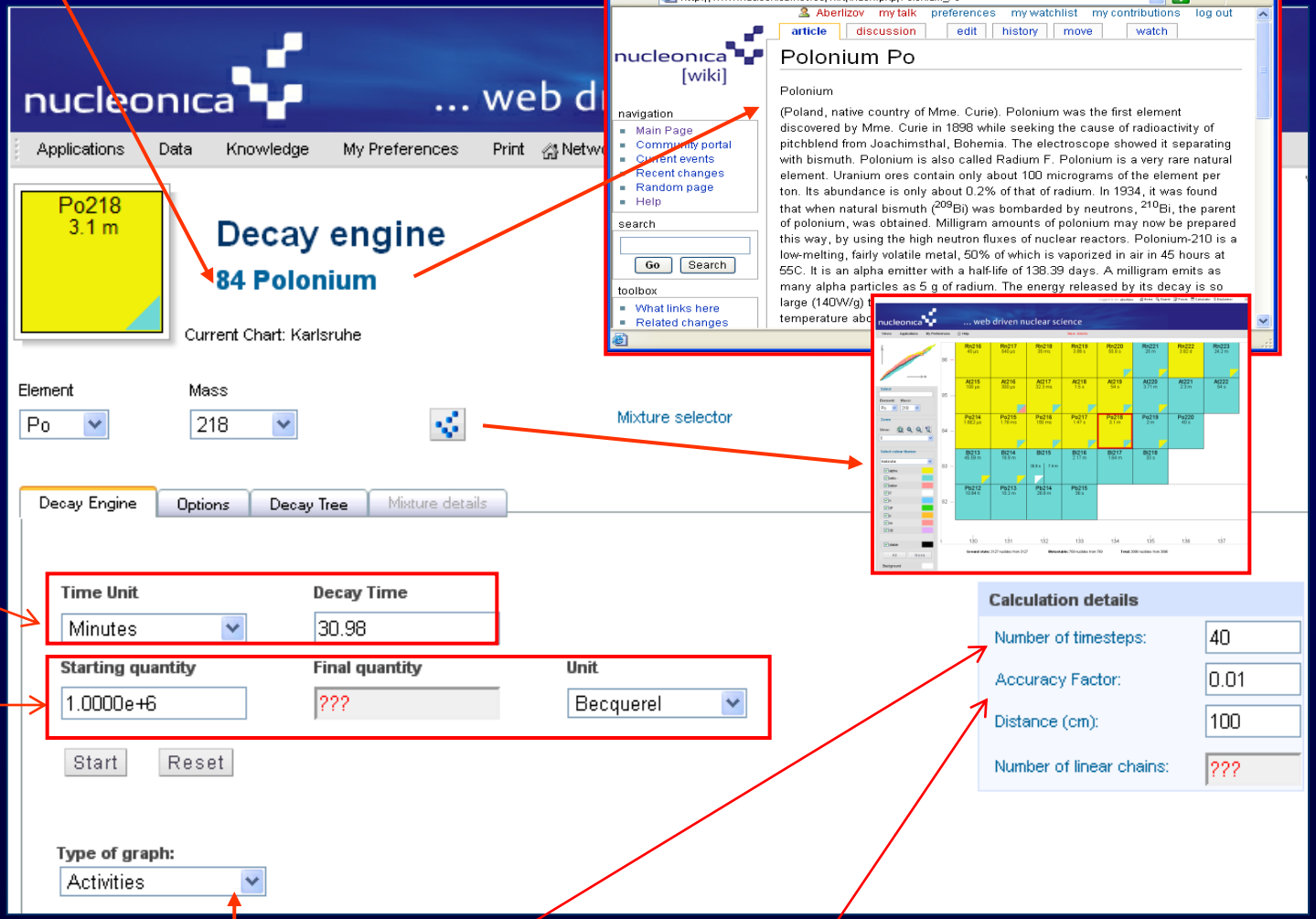
Element Info in Nucleonica wiki

Nuclide selection tools





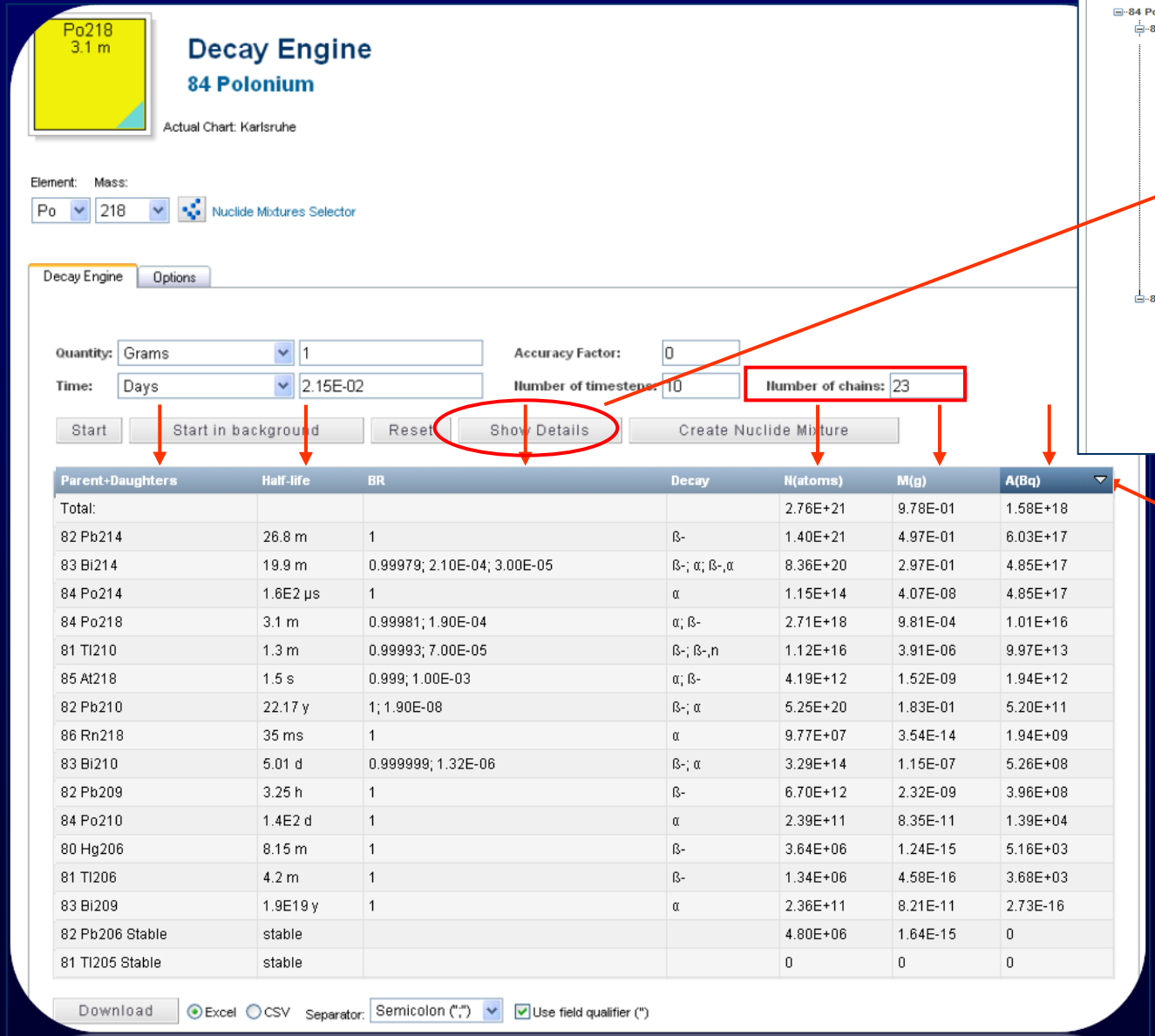
Set time

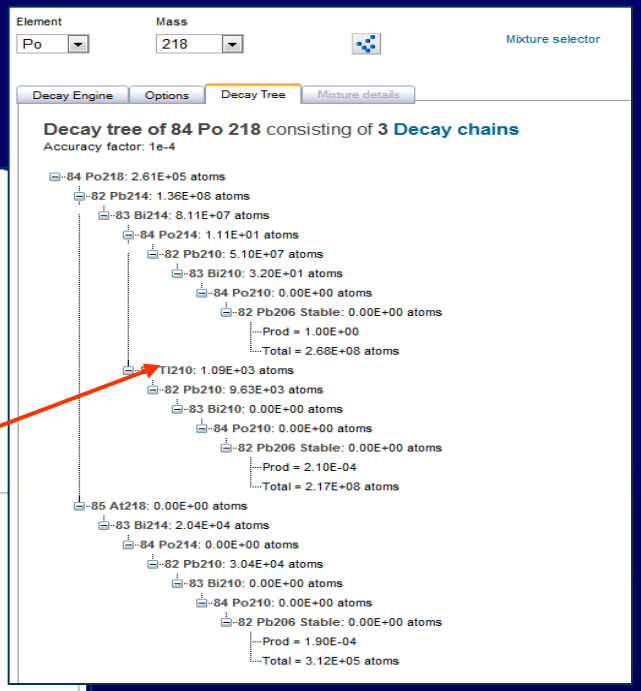
Set quantity

Select the value to be plotted on a graph

Set the number of timesteps. To plot a graph up to 40 timesteps can be used.

Set the accuracy of the calculation. Default value 1E-2 gives at least the main chain.Min. **Zero value gives all chains!**





Click on the column title to arrange the data in ascending /descending order on the parameter chosen

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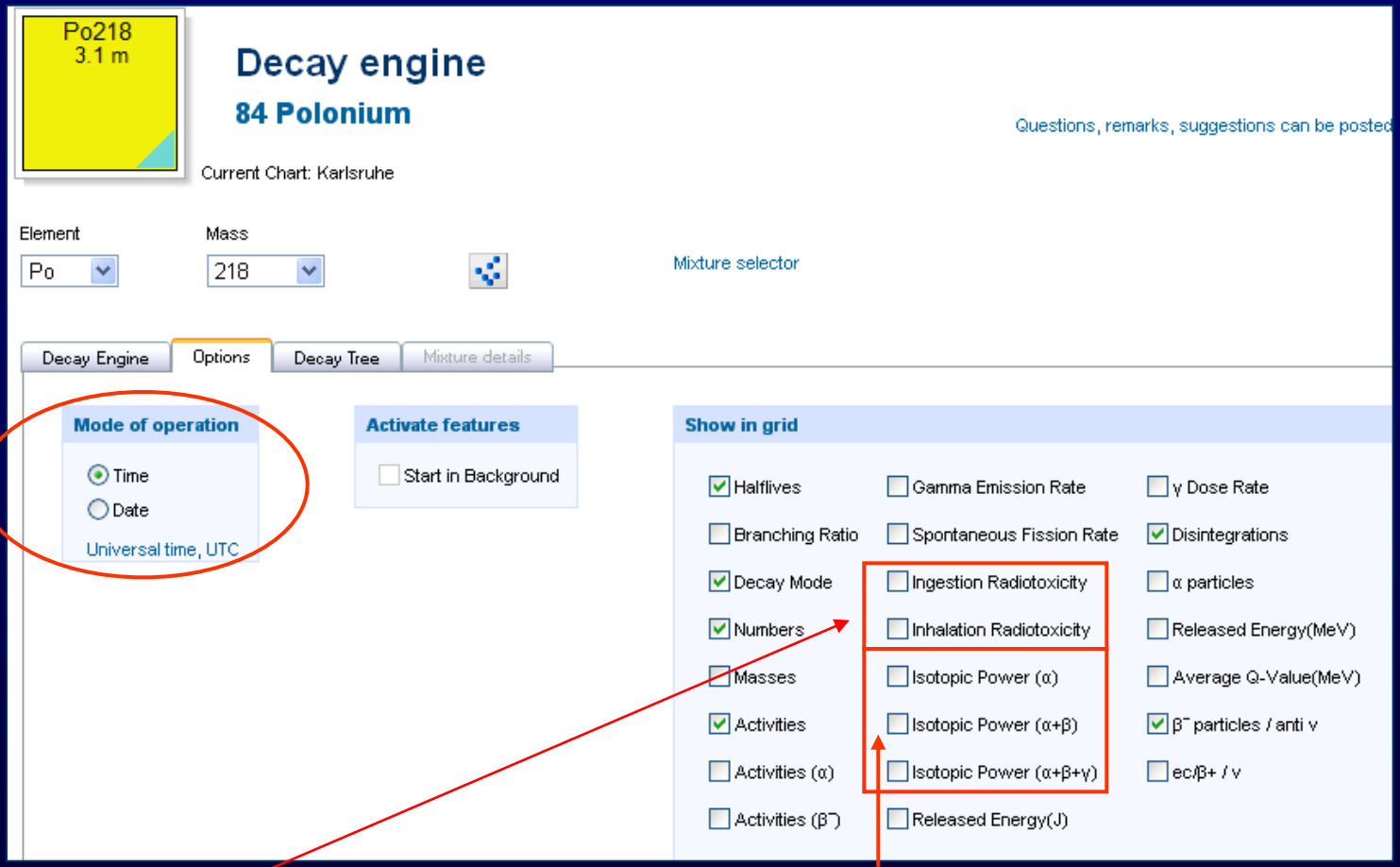

Radiotoxicity (Sv) = Activity · *e*(50), where e(50) - effective dose coefficient, which accounts for radiation and tissue weighting factors, metabolic and biokinetic information

the heat generated per unit time by the decay radiations (W)

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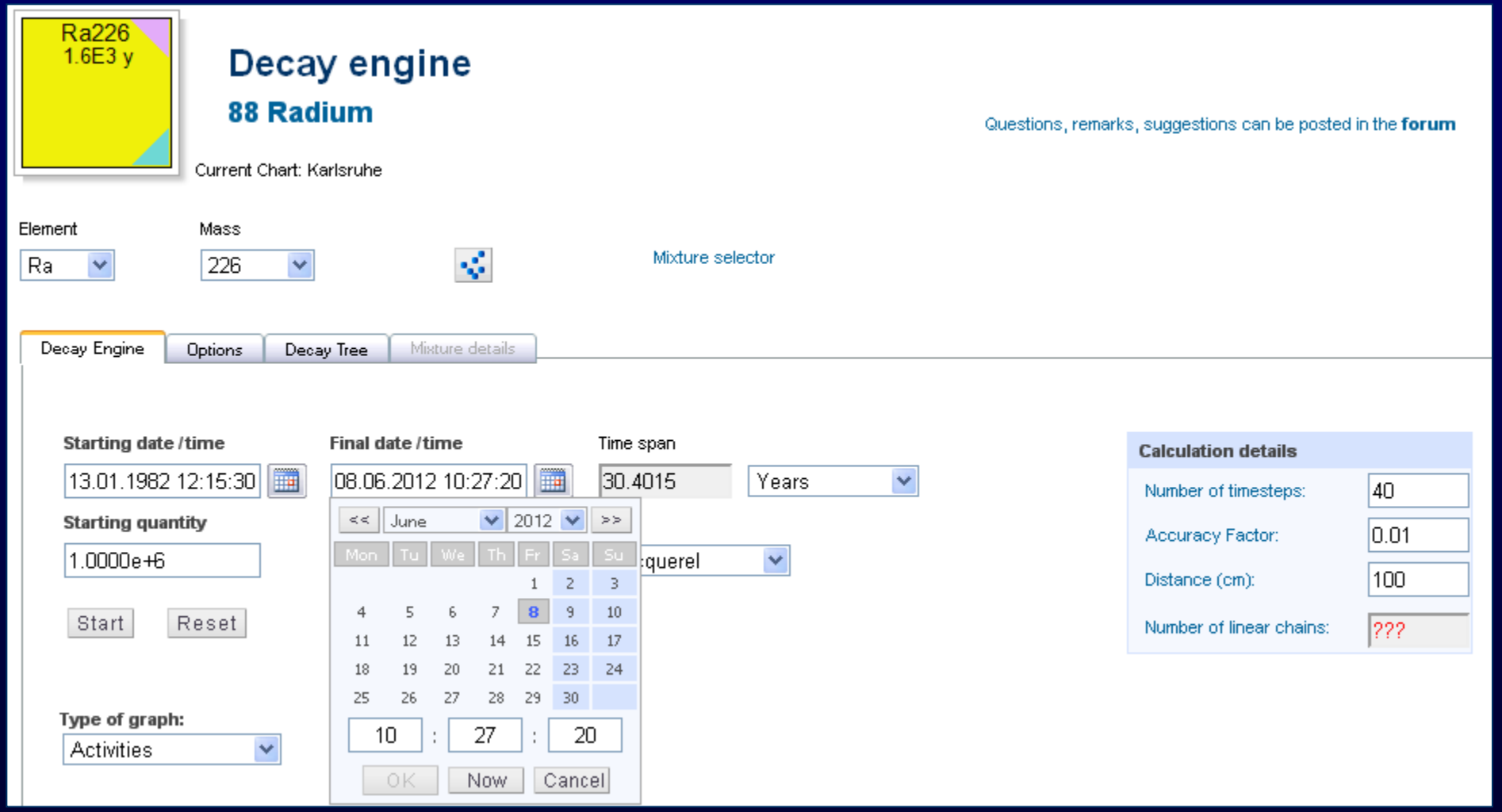
Date mode:



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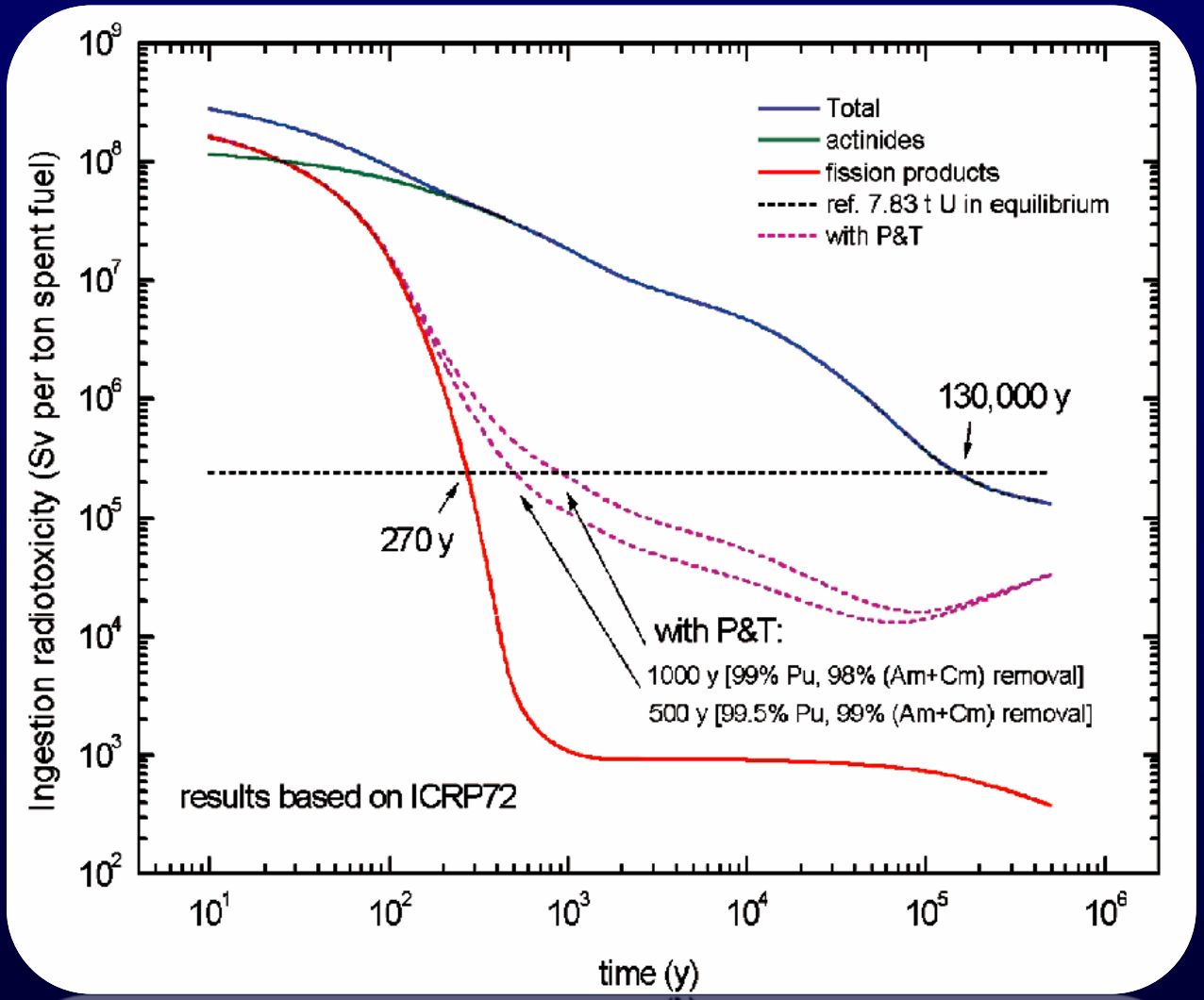
Total
actinides
fission products
ref. 7.83 t U in equilibrium
with P&T
130,000 y
270 y
with P&T:
1000 y [99% Pu, 98% (Am+Cm) removal]
500 y [99.5% Pu, 99% (Am+Cm) removal]
results based on ICRP72
Ingestion radiotoxicity (Sv per ton spent fuel)
time (y)

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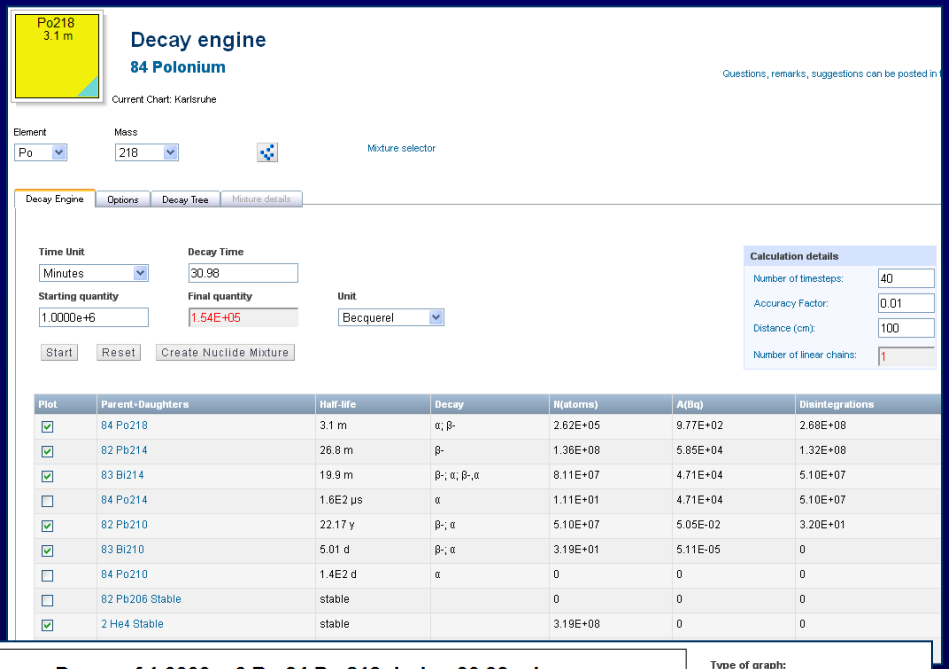
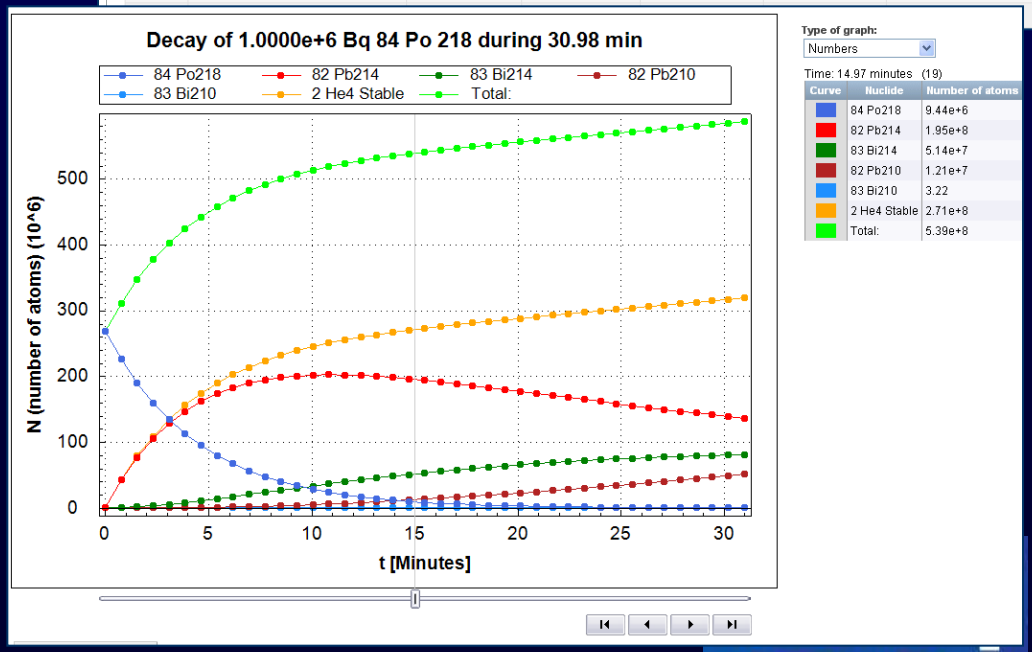
Selecting options

Plotting results





Thanks!

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